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STRESS CALCULATIONS FOR LIFETIME PREDICTION IN TURBINE BLADES

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Abstract—In order to predict fatigue lifetime in cooled turbine blades it is necessary to know stabilized cyclic stress and strain fields for typical ground-to-ground cycle.

Previous works permit to obtain the viscoplastic temperature dependent behavior and the classical one-dimensional plane cross-section method gives the stress and strain fields on the blade.

However this classical method does not take into account the restraining effects of moments due to action of centrifugal forces on the deflected blade. This paper presents an extension of the plane cross-section method in which the coupled problem induced by these restraining effects is solved without iteration.

Results are given for a turbine blade made of a refractory alloy and compared with the case in which restraining effects are neglected.

I. INTRODUCTION

In order to predict lifetime in fatigue of metallic structures at elevated temperature, classical methods involve stabilized cycle characteristics: maximum and mean stresses, viscoplastic or total strain range, energy of dissipation . . .[1–3]. Such quantities cannot be generally measured in structures, they have to be calculated; this is a difficult problem in the case of high temperature where viscoplastic behavior gives rise to a highly non-linear process.

In a previous work[4] a simple method, applicable in the elasto–viscoplastic range, has been developed in order to calculate the stress and strain fields in a turbine blade for any cyclic program of temperature and loads. The two main hypotheses are: (1) an isotropic strain-hardening behavior for the material, and (2) that a plane cross-section of the blade remains plane. This gives a one-dimensional problem which constitutes a particular case of a general three-dimensional method[5]. However this method does not take into account the restraining effects of moments due to the action of the centrifugal forces on the deflected blade.

Here the plane cross-section method is developed in order to solve without iteration the statically indeterminate problem arising by the introduction of these restraining effects.

II. CONSTITUTIVE EQUATIONS

The one-dimensional constitutive equations used here are a particular case of three-dimensional constitutive equations written by Lemaitre[6], which constitute a generalization of laws previously developed by Odqvist *et al.*; for any kind of loading, written for rates of parameters, each of them being time and x, y, z coordinates dependent:

$$\text{Partition equation:} \qquad \dot{\varepsilon} = \dot{\varepsilon}^{e} + \dot{\varepsilon}^{p} + \dot{\varepsilon}^{th} \tag{1}$$

Derived Hooke's law: $\dot{\sigma} = E\dot{\varepsilon}^e + \frac{\dot{E}}{E}\sigma$ (2)

Thermal expansion rate: $\dot{\varepsilon}^{th} = \alpha\dot{T} + \dot{\alpha}T$ (3)

Viscoplastic law: $\dot{\varepsilon}^p = h(\bar{\varepsilon}^p, \sigma)\sigma$ (4)

where σ is the stress, ε, ε^e, ε^p, ε^{th} are respectively the total, elastic, viscoplastic and thermal strains, T is the temperature, α is the linear thermal expansion coefficient, and E is the Young's modulus; h is a non-linear function which is temperature dependent. $\bar{\varepsilon}^p$ is the work-hardening-cumulant parameter which can be taken as

$$\bar{\varepsilon}^p = \int_0^t |\dot{\varepsilon}^p(\tau)| \, d\tau$$

for isotropic materials.

This parameter gives good results in the case of quasi-static loading [5,7], but in the case of cyclic loading it is only approximate.

III. METHOD OF CALCULATION

1. *General*

Consider a cooled turbine blade on which thermal gradients are important and are functions of time for the ground-to-ground cycle; centrifugal forces and aerodynamic loading are acting also as functions of time: vibration effects are neglected. Then we are dealing with a quasi-static problem for one cycle. Twist and torque on the blade are neglected.

Classical blade compensation is made for maximum stress regime[8] with resultant moments that are small or zero. However, thermal gradients induce bending of the blade, out of the compensation line, and it is necessary to take into account the restraining moments induced by centrifugal forces.

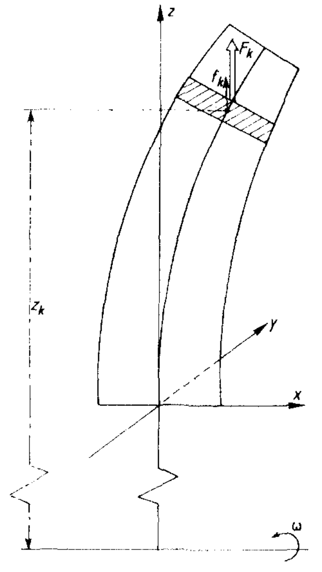

Fig. 1. Blade configuration.

Stress calculations on a blade without centrifugal restraining moments is classical, by superposition of many cross-sections by means of the plane cross-section method[2,9]. Two extreme cases (no moments or no cross-section rotation) show a big difference on stress values and emphasize the necessity of taking into account the restraining moments[4].

A coupled problem arises because restraining moments are functions of deflections, and we could conclude that an iterative process is needed. One of the purposes of the method described below is to solve the problem without iteration, which, in a viscoplastic problem, saves computer time.

2. *Method description*

2.1 *Displacement calculation.* The assumption is made that plane sections remain plane, but rotation is allowed. In x, y, z coordinates (see Fig. 1) we can write:

$$\varepsilon(x, y, z) = A(z) + B(z)x + C(z)y$$

where $A(z)$ is the strain along the neutral axis, $B(z)$ and $C(z)$ are the curvatures respectively in the x–z and y–z planes. The blade is split up into n equidistant cross-sections, each of them being decomposed into independent z axis elements, and we suppose that B and C vary as linear functions from one cross-section to the following:

$$z_k \leqslant z \leqslant z_{k+1} \Rightarrow \begin{cases} B(z) = B_k + \dfrac{B_{k+1} - B_k}{\rho}(z - z_k) \\ \\ C(z) = C_k + \dfrac{C_{k+1} - C_k}{\rho}(z - z_k) \end{cases}$$

with $\rho = z_{k+1} - z_k$.

In order to find the displacements $u(z)$ and $v(z)$ of the neutral axis, we use the fundamental beam equation:

$$u''(z) = -B(z)$$
$$v''(z) = -C(z)$$

(the sign $-$ is due to the definition of B and C in the plane cross-section hypothesis).

Integrating twice between z_{k-1} and z_k we obtain:

$$u'_k = u'_{k-1} - \frac{\rho}{2}(B_k + B_{k-1})$$

$$u_k = u_{k-1} + \rho u'_{k-1} - \frac{\rho^2}{6}(B_k + 2B_{k-1}).$$

A double recurrence gives u_k as a linear function of the curvatures:

$$u_1 = -\frac{1}{6}\rho^2 B_1 \qquad u_k = -\frac{1}{6}\rho^2 B_k - \rho^2 \sum_{j=1}^{k-1}(k - j)B_j.$$

In order to solve the problem, we have substituted the first term of u_k by $-\frac{1}{6}\rho^2 B_1$. This gives a small error which can be considered as a second order error in regards of initial hypothesis. Proceeding in the same way for v_k we have finally:

$$u_1 = -\frac{1}{6}\rho^2 B_1 \qquad u_k = u_1 - \rho^2 \sum_{j=1}^{k-1}(k-j)B_j$$
$$v_1 = -\frac{1}{6}\rho^2 C_1 \qquad v_k = v_1 - \rho^2 \sum_{j=1}^{k-1}(k-j)C_j. \tag{5}$$

2.2 *Force and moments calculation.* We assume that the mass of each cross-section is concentrated on its gravity center. Force F_i acting on the ith cross-section results from centrifugal forces induced by each mass m_k, for $k > i$:

$$F_i = \sum_{k=i+1}^{n} f_k = \sum_{k=i+1}^{n} m_k \omega^2 z_k. \tag{6}$$

In order to calculate moments acting on the ith cross-section we neglect displacement in regard to the z axis. Forces contributing to the moments on each cross-section are (g_k^1, g_k^2, f_k), where g_k^1 and g_k^2 are aerodynamic loads.

$$M_i^1 = \sum_{k=i+1}^{n} [(v_k - v_i)f_k - (z_k - z_i)g_k^2)$$
$$M_i^2 = \sum_{k=i+1}^{n} [(z_k - z_i)g_k^1 - (u_k - u_i)f_k]$$
$$M_i^3 = 0 \quad \text{(Torque neglected).}$$

Then for instantaneous variations:

$$\dot{M}_i^1 = \sum_{k=i+1}^{n} (\dot{v}_k - \dot{v}_i)f_k + \sum_{k=i+1}^{n} [(v_k - v_i)\dot{f}_k - (z_k - z_i)\dot{g}_k^2]$$
$$\dot{M}_i^2 = -\sum_{k=i+1}^{n} (\dot{u}_k - \dot{u}_i)f_k - \sum_{k=i+1}^{n} [(u_k - u_i)\dot{f}_k - (z_k - z_i)\dot{g}_k^1]. \tag{7}$$

The first terms of second members of these equations can be rederived with equations (5):

$$\sum_{k=i+1}^{n} (\dot{v}_k - \dot{v}_i)f_k = -\sum_{j=1}^{n-1} \Lambda_{ij}\dot{C}_j$$
$$-\sum_{k=i+1}^{n} (\dot{u}_k - \dot{u}_i)f_k = \sum_{j=1}^{n-1} \Lambda_{ij}\dot{B}_j \tag{8}$$

with $\Lambda_{ij} = \Lambda_{ji} = \rho^2 \sum_{k=i+1}^{n} (k-i)f_k$.

2.3 *One cross-section equilibrium (ith cross-section).*
—*Quasi static equilibrium:*

$$\int_{s_i} \dot{\sigma}\, \mathrm{d}s = \dot{F}_i \qquad \int_{s_i} \dot{\sigma} y\, \mathrm{d}s = \dot{M}_i^1 \qquad \int_{s_i} \dot{\sigma} x\, \mathrm{d}s = -\dot{M}_i^2 \tag{9}$$

s_i being the i cross-section area.
—*Plane cross-section hypothesis:*

$$\dot{\varepsilon} = \dot{A}_i + \dot{B}_i x + \dot{C}_i y. \tag{10}$$

Matching this equation with constitutive equations (1–3) gives:

$$\dot{\sigma} = E(\dot{A}_i + \dot{B}_i x + \dot{C}_i y - \dot{\varepsilon}^p - \alpha\dot{T} - \dot{\alpha}T) + \frac{\dot{E}}{E}\sigma \tag{11}$$

introducing in equation (9) we obtain the system:

$$\begin{bmatrix} K_{11}^i & K_{12}^i & K_{13}^i \\ K_{12}^i & K_{22}^i & K_{23}^i \\ K_{13}^i & K_{23}^i & K_{33}^i \end{bmatrix} \times \begin{bmatrix} \dot{A}_i \\ \dot{B}_i \\ \dot{C}_i \end{bmatrix} = \begin{bmatrix} \dot{F}_i \\ -\dot{M}_i^2 \\ \dot{M}_i^1 \end{bmatrix} + \int_{s_i} \left[E(\dot{\varepsilon}^p + \alpha\dot{T} + \dot{\alpha}T) - \frac{\dot{E}}{E}\sigma\right] \begin{bmatrix} 1 \\ x \\ y \end{bmatrix} \mathrm{d}S \tag{12}$$

where 3×3 matrix K^i is:

$$K_i = \int_{s_i} E \begin{bmatrix} 1 & x & y \\ x & x^2 & xy \\ y & xy & y^2 \end{bmatrix} \mathrm{d}S.$$

Coupling appears in the first column of the second member which contains $\{\dot{B}_j, \dot{C}_j\}_{j=1}^{n-1}$ as unknowns. Writing system (12) for each cross-section it is possible to transfer the unknowns to the first member, which avoids iteration. Then we obtain the following $3n-3$ order symmetrical system:

$$\left[\begin{array}{ccc|ccc|ccc} K_{11}^1 & & & K_{12}^1 & & & K_{13}^1 & & \\ & \ddots & & & \ddots & & & \ddots & \\ & & K_{11}^{n-1} & & & K_{12}^{n-1} & & & K_{k3}^{n-1} \\ \hline K_{12}^1 & & & & & & K_{23}^1 & & \\ & \ddots & & & \Phi & & & \ddots & \\ & & K_{12}^{n-1} & & & & & & K_{23}^{n-1} \\ \hline K_{13}^1 & & & K_{23}^1 & & & & & \\ & \ddots & & & \ddots & & & \Psi & \\ & & K_{13}^{n-1} & & & K_{23}^{n-1} & & & \end{array}\right] \times \begin{bmatrix} \dot{A}_1 \\ \vdots \\ \dot{A}_{n-1} \\ \hline \dot{B}_1 \\ \vdots \\ \dot{B}_{n-1} \\ \hline \dot{C}_1 \\ \vdots \\ \dot{C}_{n-1} \end{bmatrix} = \begin{bmatrix} P_1 \\ \vdots \\ P_{n-1} \\ \hline Q_1 \\ \vdots \\ Q_{n-1} \\ \hline R_1 \\ \vdots \\ R_{n-1} \end{bmatrix}$$

Φ and Ψ matrices are (with δ_{ij} Kronecker delta symbol):

$$\Phi_{ij} = K_{22}^i \delta_{ij} + \Lambda_{ij} \qquad \Psi_{ij} = K_{33}^i \delta_{ij} + \Lambda_{ij}.$$

The load vectors are:

$$P_i = \sum_{k=i+1}^{n} \dot{f}_k + \int_{s_i} \left[E(\dot{\varepsilon}^p + \alpha\dot{T} + \dot{\alpha}T) - \frac{\dot{E}}{E}\sigma\right] \mathrm{d}S$$

$$Q_i = -\sum_{k=i+1}^{n} [(u_k - u_i)\dot{f}_k - (z_k - z_i)\dot{g}_k^1] + \int_{s_i} \left[E(\dot{\varepsilon}^p + \alpha\dot{T} + \dot{\alpha}T) - \frac{\dot{E}}{E}\sigma\right] x\, \mathrm{d}S$$

$$R_i = \sum_{k=i+1}^{n} [(v_k - v_i)\dot{f}_k - (z_k - z_i)\dot{g}_k^2] + \int_{s_i} \left[E(\dot{\varepsilon}^p + \alpha\dot{T} + \dot{\alpha}T) - \frac{\dot{E}}{E}\sigma\right] y\, \mathrm{d}S.$$

This system can easily be reduced to an $n-1$ order system.

2.4. *Step-by-step linearization procedure.* The problem is solved by linearization at each step:

—At time t the following quantities are given: rotational speed, aerodynamic loads, temperatures, and the derivatives of all these terms. Constitutive material characteristics are known as functions of temperature.

—At time t stress and viscoplastic strain fields are known for each cross-section from the preceding calculations.

—Viscoplastic constitutive equation (4) gives the viscoplastic strain rate field, then the second member of system (13) can be evaluated, which gives $\{\dot{A}_j, \dot{B}_j, \dot{C}_j\}_{j=1}^{n-1}$. Instantaneous variations of displacements and stresses are easily deduced by the equations (5) and (11).

—From t to $t+\Delta t$ the first order Taylor's formulae are applied, Δt being automatically chosen so as to limit the error due to the neglect of second order terms[10]: hence this step is small when the viscoplastic strain rate is rapidly varying and is very large in the stationary case.

—The first step, arbitrarily small, is calculated as elastic for the stresses.

IV. APPLICATION AND RESULTS

This method has been applied to a cooled turbine blade made of a refractory alloy. The viscoplastic law was particularized by:

$$\dot{\varepsilon}^p = \left[\frac{|\sigma|}{K\bar{\varepsilon}_p^{\,1/m}}\right]^n \frac{\sigma}{|\sigma|}$$

where n, m, K were respectively viscosity, strain-hardening and strength parameters. These parameters were experimentally determined, between 500–1100°C, by relaxation, constant strain rate and creep tests. The law has been validated by comparisons made between experimental and theoretical results in complex tension compression tests with time dependent temperature.

The temperature field on the blade and its evolution for a ground-to-ground cycle is given by calculations[11]. They are shown in Fig. 2.

Calculations are made successively for 4, 6, 8, 10 superposed and coupled cross-sections, each of them being decomposed into 140 elements. The deflections obtained at a given time are presented in Fig. 3. We can see a good correlation for convergence with a minimum number of cross-sections, and the difference with calculations made without centrifugal restraining effects.

The strain range quantities required as inputs to a failure criteria were calculated by the two methods. Figure 4 demonstrates the results by means of contour plots of these values on a typical blade planform and Figs. 5 and 6 show variations of these quantities along the radial axis of the blade (z coordinate).

One can see that the results of calculations made without restraining effects, are not as satisfactory as regards security.

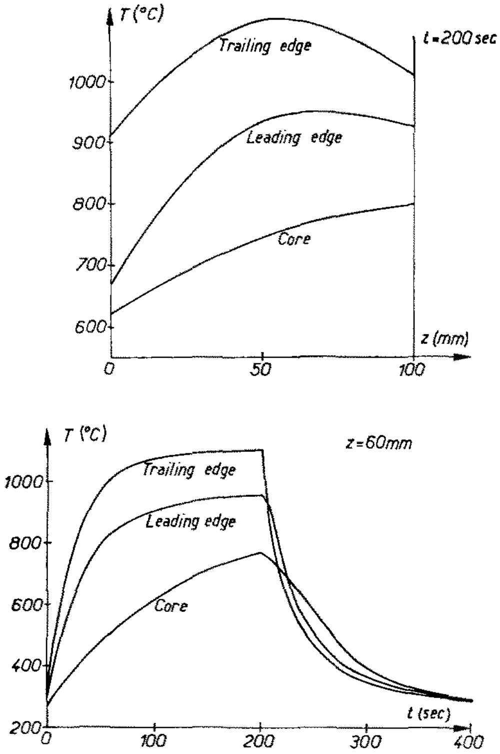

Fig. 2. Temperature distribution and evolution.

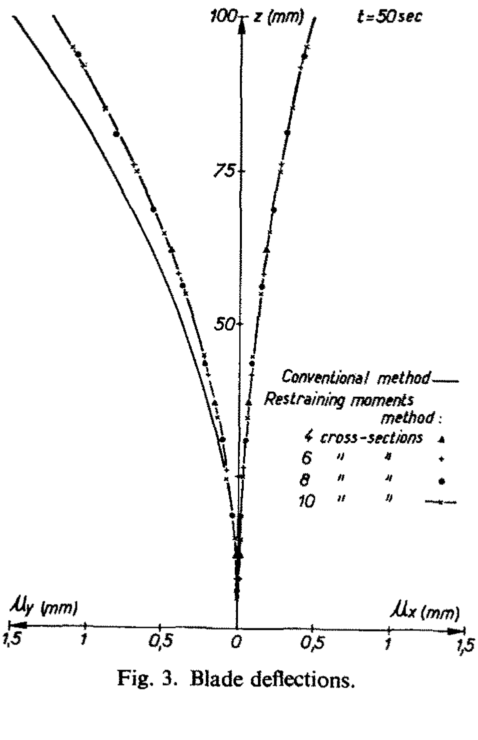

Fig. 3. Blade deflections.

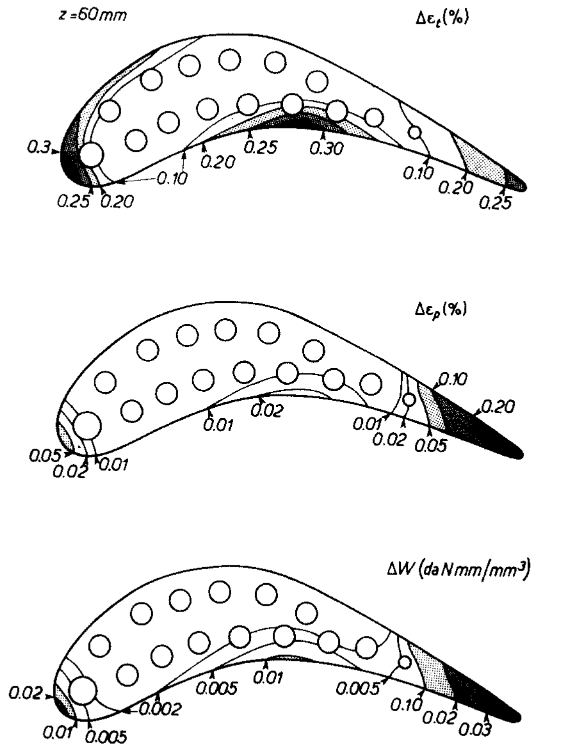

Fig. 4. Fatigue parameter distribution for one cross-section:
$\Delta\varepsilon_t$: total mechanical strain range.
$\Delta\varepsilon_p$: viscoplastic strain range.
ΔW: variation of viscoplastic energy density:
$\Delta W = \int_{\text{cycle}} \sigma(\tau)\dot{\varepsilon}^p(\tau)\,d\tau$.

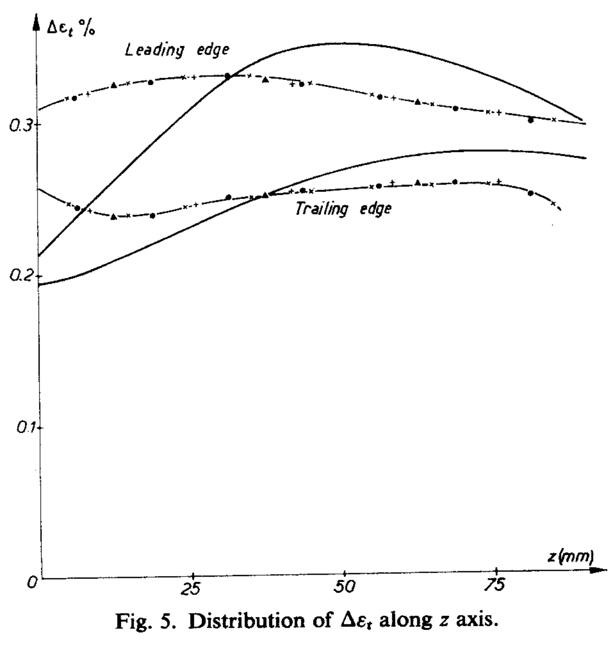

Fig. 5. Distribution of $\Delta\varepsilon_t$ along z axis.

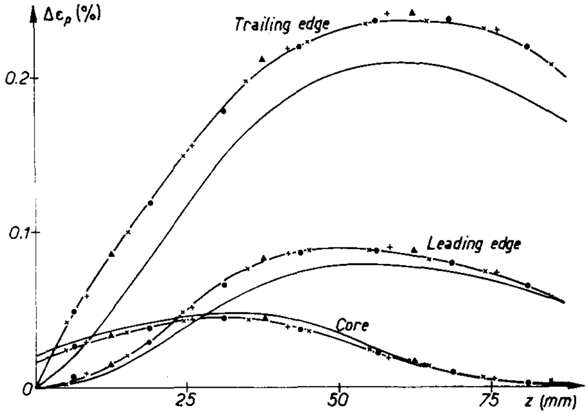

Fig. 6. Distribution of $\Delta\varepsilon_p$ along z axis.

V. CONCLUSION

These results show that the influence of the centrifugal restraining moments is important for the knowledge of the precise stress and strain history and of the quantities involved in fatigue criteria. For instance, the works of Manson, Coffin and other authors show that a viscoplastic strain range increase of about 20 per cent could produce a lifetime diminution of 40 or 50 per cent.

Hence, in further works with more sophisticated codes and more complicated laws, it would be interesting and important to take into account these restraining effects.

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Абстракт — В целью предсказания долговечности усталости в охлажденных лопатках турбины является нужным знание полей стабилизированных циклических напряжений и деформаций для типичного уземленного цикла.

Предыдущие работы дают возможность получить вязкопластическое поведение в зависимости от температуры. Метод классического одномерного плоского разреза определяет поля напряженний и деформации в лопатке.

Этот классический метод, однако, не учитывает эффектов ограничения момента, вызванного действием центробежных сил на деформированную лопатку. Предлагаемая работа дает обобщение метода плоского разреза, в котором сопряженная задача, вызванная этими эффектами ограничения, решена без помощи итерации.

Даются результаты для лопатки турбины, изготовленной из жаростойкого сплава и сравниваются со случаем, для которого эффекты ограничения пренебрегаются.